

CBCS SCHEME

USN

--	--	--	--	--	--	--	--	--	--

18MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform of:
- (i) $\left(\frac{4t+5}{e^{2t}}\right)^2$ (ii) $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$ (iii) $t \cos at$. (10 Marks)
- b. The square wave function $f(t)$ with period $2a$ defined by $f(t) = \begin{cases} 1 & 0 \leq t < a \\ -1 & a \leq t < 2a \end{cases}$. Show that $\left(\frac{1}{s}\right) \tanh\left(\frac{as}{2}\right)$. (05 Marks)
- c. Employ Laplace transform to solve $\frac{d^2y}{dt^2} - \frac{dy}{dt} = 0$, $y(0) = y_1(0) = 3$. (05 Marks)

OR

- 2 a. Find (i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ (ii) $\cot^{-1}\left(\frac{s}{2}\right)$ (iii) $L^{-1}\left\{\frac{s}{(s+2)(s+3)}\right\}$ (10 Marks)
- b. Find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$ using convolution theorem. (05 Marks)
- c. Express $f(t) = \begin{cases} 2 & \text{if } 0 < t < 1 \\ \frac{t^2}{2} & \text{if } 1 < t < \frac{\pi}{2} \\ \cos t & t > \frac{\pi}{2} \end{cases}$ in terms of unit step function and hence find its Laplace transformation. (05 Marks)

Module-2

- 3 a. Obtain the Fourier series of $f(x) = \begin{cases} 2 & -2 < x < 0 \\ x & 0 < x < 2 \end{cases}$. (08 Marks)
- b. Find the half range cosine series of, $f(x) = (x+1)$ in the interval $0 \leq x \leq 1$. (06 Marks)
- c. Express $f(x) = x^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

f'ega kaal

OR

- 4 a. Compute the first two harmonics of the Fourier Series of
- $f(x)$
- given the following table :

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

(08 Marks)

- b. Find the half range size series of
- e^x
- in the interval
- $0 \leq x \leq 1$
- .

(06 Marks)

- c. Obtain the Fourier series of
- $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$
- valid in the interval
- $(-\pi, \pi)$

(06 Marks)

Module-3

- 5 a. Find the Infinite Fourier transform of $e^{-|x|}$. (07 Marks)
- b. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$. (06 Marks)
- c. Solve $u_{n+2} - 3u_{n+1} + 2u_n = 3^n$, given $u_0 = u_1 = 0$. (07 Marks)

OR

- 6 a. If
- $f(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$
- , find the infinite transform of
- $f(x)$
- and hence evaluate
- $\int_0^{\infty} \frac{\sin x}{x} dx$
- .

(07 Marks)

- b. Obtain the Z-transform of
- $\cosh n\theta$
- and
- $\sinh n\theta$
- .

(06 Marks)

- c. Find the inverse Z-transform of
- $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$

(07 Marks)

Module-4

- 7 a. Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's Series method upto 4th degree terms and find the value of $y(1.1)$. (07 Marks)
- b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} + y = 2x$ at $x = 1.1$ given $y(1) = 3$ (Take $h = 0.1$). (06 Marks)
- c. Apply Milne's predictor-corrector formulae to compute $y(0.4)$ given $\frac{dy}{dx} = 2e^x y$, with (07 Marks)

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

OR

- 8 a. Given $\frac{dy}{dx} = x + \sin y$, $y(0) = 1$. Compute $y(0.4)$ with $h = 0.2$ using Euler's modified method. (07 Marks)
- b. Apply Runge-Kutta fourth order method, to find $y(0.1)$ with $h = 0.1$ given $\frac{dy}{dx} + y + xy^2 = 0$; $y(0) = 1$. (06 Marks)
- c. Using Adams-Bashforth method, find $y(4.4)$ given $5x \left(\frac{dy}{dx} \right) + y^2 = 2$ with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

(07 Marks)

Module-5

- 9 a. Solve by Runge Kutta method $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for $x = 0.2$ correct 4 decimal places, using initial conditions $y(0) = 1, y'(0) = 0, h = 0.2$. (07 Marks)
- b. Derive Euler's equation in the standard form, $\frac{\partial f}{\partial y} - \frac{d}{dx} \left[\frac{\partial f}{\partial y'} \right] = 0$. (06 Marks)
- c. Find the extremal of the functional, $\int_{x_1}^{x_2} y^2 + (y')^2 + 2ye^x dx$. (07 Marks)

OR

- 10 a. Apply Milne's predictor corrector method to compute $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values:

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

(07 Marks)

- b. Find the extremal for the functional, $\int_0^{\frac{\pi}{2}} [y^2 - y'^2 - 2y \sin x] dx$; $y(0) = 0$; $y\left(\frac{\pi}{2}\right) = 1$. (06 Marks)
- c. Prove that geodesics of a plane surface are straight lines. (07 Marks)
